Exam Statistical Physics

Friday, January 24, 2014

The total number of points is 50. Do not forget to write your name and the student number on the first sheet. Some useful constants and integrals can be found below the last problem. Good luck.

1. **Microcanonical ensemble**

   Consider a gas with the statistical weight

   $$\Omega(E) = Be^{AE^{3/4}V^{1/4}},$$

   where E and V are, respectively, the energy and volume of the gas, and A and B are some coefficients.

   (a) Find the energy of the gas at a temperature T: $E = E(T,V)$. [4 points]

   (b) Calculate the pressure, $P = P(T,V)$, of the gas and find a relation between P and the energy density $\frac{E}{V}$. [4 points]

   (c) What gas has these properties? [2 points]

2. **Ideal classical gas**

   Consider an ideal classical gas of N atoms at a temperature T. The gas occupies the volume V. The atom mass is m.

   (a) Show that the single-atom partition function, $z = \sum_{\mathbf{p}} e^{-\beta\varepsilon_{\mathbf{p}}}$, is given by

   $$z = \frac{V}{\lambda^3},$$

   where $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$ is the termal de Broglie wave length. [5 points]

   (b) Show that the entropy of the gas is given by the following expression,

   $$S = Nk_B\left[\ln\left(\frac{V}{N\lambda^3}\right) + \frac{5}{2}\right].$$

   [5 points]

   Hint: Find the free energy of the gas F.

3. **Specific heat of ideal gas**

The table below shows the ratios of specific heat at constant pressure and at constant volume, c_p/c_v, for various gases measured at 1 atm and 20 °C.

gas	He	Ne	Ar	H_2	CO	air	H_2S	NH_3	CH_4
c_p/c_v	1.67	1.67	1.67	1.41	1.40	1.40	1.32	1.31	1.30

Explain these values and express c_p/c_v in terms of physically motivated simple fractions, which fit the experimental data with a few percent precision. [10 points]

Hint: What can one conclude about the rotational and vibrational temperatures and the number of the corresponding degrees of freedom in these gases?

4. **Root-mean-square velocity in classical and quantum gases**

Find the root-mean-square velocity of particles with the mass m for the following gases:

(a) Ideal classical gas at a temperature T. Calculate the rms velocity in terms of T and m. [5 points]

(b) Ideal Fermi gas of electrons at zero temperature. What is the rms velocity in terms of the electron density n and the electron mass? [4 points]

(c) Ideal Bose gas at zero temperature. [1 point] $E=0 \Rightarrow V=0.$

5. **Triple point**

The vapor pressure, in mm of Hg, of solid ammonia is given by the relation:

$$\ln P(\mathrm{mmHg}) = 23.03 - \frac{3754\mathrm{K}}{T}, \tag{1}$$

where T is the absolute temperature (in other words, Eq.(1) represents the equilibrium curve of solid and gaseous ammonia). The vapor pressure of liquid ammonia is

$$\ln P(\mathrm{mmHg}) = 19.49 - \frac{3063\mathrm{K}}{T}.$$

(a) What is the temperature T_* of the triple point? [2 points]

(b) Compute the latent heat of vaporization, $L_{\mathrm{liquid}\to\mathrm{vapor}}$, and the lattice heat of sublimation, $L_{\mathrm{solid}\to\mathrm{vapor}}$, at the triple point. Express your answer in cal/mole or J/mole. The vapor can be approximately treated as an ideal gas. The density of the vapor is negligibly small compared to that of the liquid and solid ammonia. [8 points]

Hint: Use Clapeyron-Clausius equation and equation of state for ideal classical gas. Calculations of quantities per mole are conveniently done using the gas constant $R = k_B N_A =$ $1.99\ \mathrm{cal\,mole^{-1}\,K^{-1}} = 8.31\ \mathrm{J\,mole^{-1}K^{-1}}$.

Constants, conversion factors and integrals:

$\hbar = 1.1 \times 10^{-34}$ J s $= 6.6 \times 10^{-16}$ eV s, $\quad c = 3 \times 10^8$ m s^{-1}, $\quad \hbar c = 2 \times 10^{-5}$ eV cm,

$m_e = 9.1 \times 10^{-31}$ kg, $\quad m_e c^2 = 5.1 \times 10^5$ eV, $\quad k_B = \frac{1\ \mathrm{eV}}{11606\ \mathrm{K}} = 1.38 \times 10^{-23}$ J K^{-1},

$N_A = 6.02 \times 10^{23}$, $\quad R = k_B N_A = 1.99\ \mathrm{cal\ mole^{-1}\,K^{-1}} = 8.31\ \mathrm{J\,mole^{-1}K^{-1}}$,

1 eV $= 1.6 \times 10^{-19}$ J, $\quad$ 1 cal $= 4.18$ J, $\quad$ 1 atm $= 760$ mmHg $= 1.01 \times 10^5$ N m^{-2},

$a_B = \frac{\hbar^2}{m_e e^2} = 0.53$ Å, $\quad 1\text{Å} = 10^{-8}$ cm, $\quad \mathrm{Ry} = \frac{1}{2}\frac{\hbar^2}{m_e a_B^2} = \frac{1}{2}\frac{m_e e^4}{\hbar^2} = 13.6$ eV.

$$\int_0^\infty dx x^n e^{-x} = n!, \quad \int_0^\infty dx x^{2n+1} e^{-x^2} = \frac{n!}{2}, \quad \int_{-\infty}^\infty dx x^{2n} e^{-x^2} = \sqrt{\pi}\,\frac{(2n)!}{2^{2n} n!}, \quad n = 0,1,2,\ldots,$$

$$I_n = \int_0^\infty \frac{dx x^{n-1}}{e^x - 1} = (n-1)!\,\zeta(n), \quad n = 1,2,\ldots, \quad I_3 \approx 2.4, \ I_4 = \frac{\pi^4}{15} \approx 6.5.$$